1. $a^n \times b^n = (ab)^n$

2. $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$

3. Complete the square on ax^2+bx+c

$\Rightarrow a\left(x^2+\frac{b}{a}x\right)+c$

$\Rightarrow a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}\right]+c$

$\Rightarrow a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}+c$

OR

$ax^2+bx+c = a(x+d)^2+e$

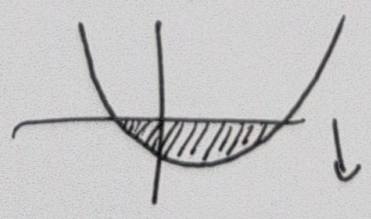
$d=\frac{b}{2a}$ $e=c-\frac{b^2}{4a}$

~~3~~4. NEVER divide by a variable, or a function of a variable or you will lose solutions. ALWAYS FACTORISE.

5) After solving non-standard simultaneous equations (logs/trig/circles), CHECK WHETHER THE SOLUTIONS WORK by substitution.

6) Quadratic inequalities:

ALWAYS SKETCH



7) Common error in proofs: multiplying or dividing by a variable in an inequality, when that variable is not defined to be positive. This flips the sign $\geq \Rightarrow \leq$

8) Use number line or a sketch for inequalities (100% success rate)

9) To find remainder, do Q-A:

$\frac{x^2+2x-4}{x-1}$

$\Rightarrow$

	x	3
x	x^2	$3x$
-1	$-x$	-3

$Q=-4$
$A=-3$
$-4--3=-1$
$\therefore$ remainder $=-1$

10) Function = quotient × divisor + remainder

OR

$\frac{\text{Function}}{\text{divisor}} = \text{Quotient} + \frac{\text{remainder}}{\text{divisor}}$

11) Comparing coefficients is faster than division.

$\frac{3x^3+4x^2-13x+6}{x+3}$

$\Rightarrow 3x^3+4x^2-13x+6 = (ax^2+bx+c)(x+3)+d$

x^3: $a=3$

x^2: $b+3a=4$
$b=4-9$
$b=-5$

x: $3b+c=-13$
$c=-13+15$
$c=2$

x^0: $3c+d=6$
$6+d=6$
$d=0$

$= 3x^2-5x+2$

12) Only one-to-one functions have an inverse.

To find inverse:

(1) Write in $y=\ldots$ form
(2) Rearrange to make x subject
(3) Swap x & y.

13) When you see a term-to-term sequence;

$x_{n+1} = \frac{23x_n-53}{5x_n+1}$

sub in values until a repetition.

* Get more from screenshots

14) Sum of arithmetic series:

$S_n = \frac{n}{2}(2a+(n-1)d)$

15) Sum to 'n' of geometric series:

$S_n = \frac{a(1-r^n)}{1-r}$

16) To converge, $|r|<1$
Treat these differently.

17) Sum to ∞ of geometric is

$S_\infty = \frac{a}{1-r}$

18) In a series/seq. question, find 2 of a, r or d & use simultaneous equations to find the answer.

19) When faced with binomial expansion with 3 terms, use brackets:

$x^6y^2 : (1+x^2+y)^6$

$= (1+(x^2+y))^6$

$= \binom{6}{5}(x^2+y)^5$

$= \binom{6}{5}\binom{5}{3}x^6y^2$

$= 6 \times \frac{5\times4}{2}$

$= 60$ //

20) You can estimate decimal expansions with binomial. 1.3^7

$= (1+3x)^7$

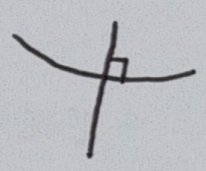
$\Rightarrow x = 0.1$

21) Midpoint of straight line is

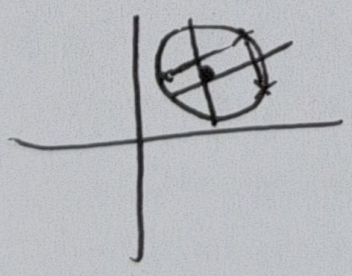
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

22) TANGENT has SAME gradient.

NORMAL has $\frac{-1}{m}$ gradient.



23) If given 3 points on a circle, find 2 perpendicular bisectors & solve simultaneously.



24) When question says 'intersections', you probably have to equate 2 equations & use discriminant / solve / differentiate.

25) CIRCLE THEOREMS !!

26) You can get 2 solutions using sine rule, * $\sin x$ & $\sin(180-x)$, if you have angle-side-side

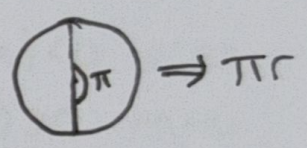
27) Cosine rule:

$a^2 = b^2+c^2-2bc\cos A$

OR $\cos A = \frac{b^2+c^2-a^2}{2bc}$

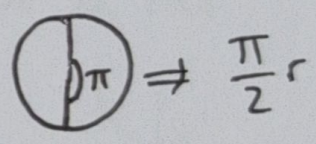
28) $180° = \pi$ rad

29) Arc length $= r\theta$



$\Rightarrow \pi r$

30) Sector area $= \frac{1}{2}r^2\theta$



$\Rightarrow \frac{\pi}{2}r^2$

31) Area of segment derived from:

Segment = sector - triangle

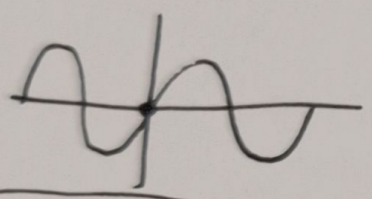
$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}r^2(\theta - \sin\theta)$

32) TRIG VALUES

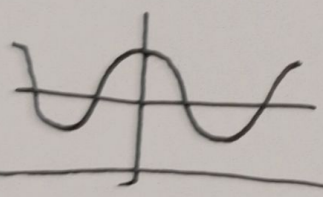
θ	30	45	60	90
θ rad	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sinθ	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cosθ	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tanθ	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N/A

33) Sine is odd function
⇒ 180° rotational symmetry around origin.



$\sin\theta = -\sin(-\theta)$

34) Cosine is even function
⇒ Symmetrical at $x = 0$



$\cos\theta = \cos(-\theta)$

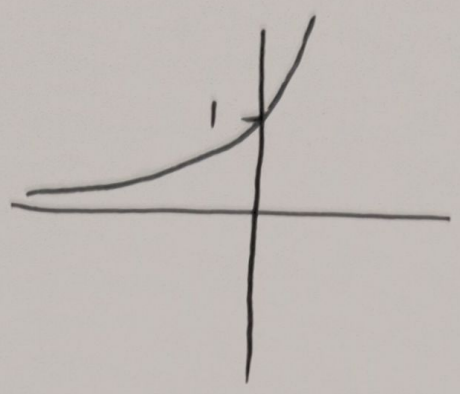
35) tan is an odd function
⇒ 180° rotational symmetry around origin.

$\tan\theta = -\tan(-\theta)$

36) Anytime you see $\sin^2\theta$ or $\cos^2\theta$, USE
$\sin^2\theta + \cos^2\theta \equiv 1$

37) Anytime you see $\tan\theta$, convert to $\frac{\sin\theta}{\cos\theta}$

38) $y = a^x$



39) $\log_a b = \frac{\log_x b}{\log_x a}$

40) $\log_a x + \log_a y = \log_a(xy)$

41) $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$

42) $k\log_a x = \log_a x^k$

43) When you see any weird things where you might be able to form simultaneous eq, USE A substitution.

$5^x = 4$,
$\cos(2x) = x$
$\log_2 p = z$

44) Increasing function:
$f'(x) > 0$

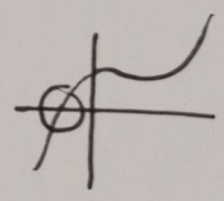
Decreasing function
$f'(x) < 0$

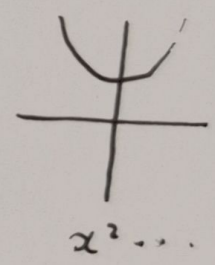
.....

IN PRACTICE:
1) differentiate function
2) = 0, solve
3) Sketch differentiated function
4) find where
>0 / <0
↓ increasing ↓ decreasing

45) MINIMA/MAXIMA
If $f''(x) < 0$ MAX
If $f''(x) > 0$ MIN

46) Every odd-order polynomial MUST have a root





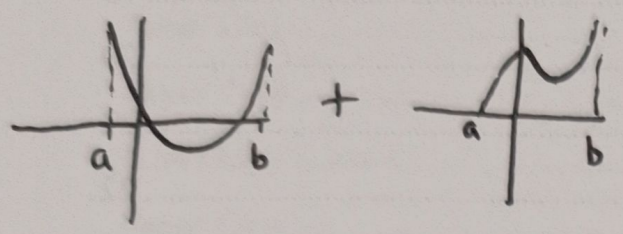
47) When doing indefinite integration, remember:

$\int x^2 = \frac{x^3}{3} + C$

48) INTEGRATION TRICKS

$$\int_b^a f(x)\,dx + \int_b^a g(x)\,dx$$

$$= \int_b^a [f(x)+g(x)]\,dx$$



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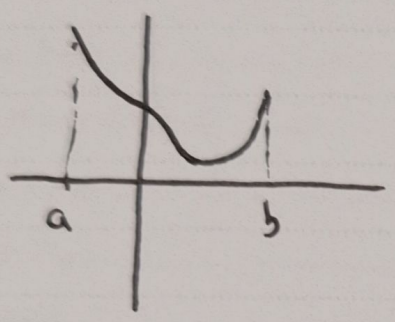

49) $\int_b^a af(x)\,dx =$

$$a\int_b^a f(x)\,dx$$

[SAVES SO MUCH TIME]

↪ Constants can be taken out of integrals

50) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

If negative, swap limits & add minus sign.

51) $\int_b^a f(x)\,dx + \int_b^c f(x)\,dx$

$$= \int_c^a f(x)\,dx$$

52) TRAPEZIUM RULE

$$\int_a^b f(x)\,dx \approx$$

$$\frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$$

$$\Rightarrow h = \frac{b-a}{n}$$

53) Overestimate for convex

$$f''(x) > 0$$

Underestimate for concave

$$f''(x) < 0$$

54) Combining odd & even functions:

Even × Even = Even

Odd × Odd = Even

Odd × Even = Odd

55) When evaluating numbers to a power exactly, use BINOMIAL:

$$\left(\frac{19}{20}\right)^n = \left(1 - \frac{1}{20}\right)^n$$

56) When square-rooting in a proof, remember the $\pm$

57) If $\sin x = \sin\alpha$

x is not necessarily α

$(30°, 150°)$

58) Be careful when making substitutions:

$3^{2x+1} \Rightarrow 3^x = 4$

$= 34^2$

59) To estimate roots (which is smallest / largest?)

⇒ square them & compare with squares of other options.

$\left(\frac{\sqrt{3}}{2}\right)^2$ vs $\left(\frac{2\sqrt{5}}{5}\right)^2$

$\frac{3}{4}$ $\quad\quad$ $\frac{20}{25} = \frac{4}{5}$

BIGGER ↑

60) If you get an answer & the negative of the answer is an option, TRIPLE CHECK any working (spend 2 extra mins JUST on this)

61) When dealing with 'power stalks', start from TOP

$$1!\left(\tfrac{1}{2}\right)^{2!\left(\frac{1}{2}\right)^{3!\left(\frac{1}{2}\right)^{4!\left(\frac{1}{2}\right)}}}$$

START HERE

62) How many ways of arranging MISSISSIPPI?

$$\frac{11!}{4!4!2!} \Rightarrow \frac{\text{letters!}}{\text{repeats!}\,\text{repeats!}\,\text{repeats!}}$$

63) Apply log change base rule:

$$\log_a b = \frac{\log_c b}{\log_c a}$$

$$\log_4 8 = \frac{\log_2 8}{\log_2 4} = \frac{3}{2}$$

64) What is the sum of the ~~possible values of~~ solutions of:

$$c^2 - 6c + 8 = 0$$

↑ it is just negative of this, so 6.

$$(c-4)(c-2)$$

$$c = 2, 4$$

$$= 6$$