TMUA PREPARATION

PAPER 2

TMUA PAPER 2

Logic Decoder

Your Complete Guide to TMUA Reasoning Questions

MASTER TECHNIQUE

"Convert to if...then form — the universal translator for every necessary, sufficient, and logic question."

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Contents

1 Why Paper 2 Feels Different

If you've looked at a TMUA Paper 2 and felt confused, you're not alone. Every year, students tell us the same thing: "*I've never seen questions like this before.*"

There's a good reason for that. Paper 2 tests a style of mathematical reasoning that simply isn't covered at A-Level. While Paper 1 mostly asks you to *calculate*, Paper 2 mostly asks you to *judge arguments and statements*. It tests whether you can:

- Understand what mathematical statements actually *mean*
- Spot when an argument is valid—and when it's subtly wrong
- Translate between different ways of saying the same thing
- Construct your own chains of logical reasoning

Here's the kind of thing you'll face:

> *"Consider the statement: 'Being prime is necessary for being odd.' Is this true or false? If false, give a counterexample."*

The good news? There's a systematic approach that works for almost every Paper 2 logic question. We call it the **"If. . . Then" Method**, and it's what this guide is all about.

Key Insight

Most Paper 2 logic questions can be solved by converting the given statements into "if. . . then" form. Once you do this, the answer often becomes obvious.

What This Guide Covers

By the end of this guide, you'll be able to:

1. Convert any statement about "necessary" or "sufficient" conditions into clear "if. . . then" form
2. Instantly identify the converse and contrapositive of any statement
3. Negate statements involving "and", "or", "for all", and "there exists"
4. Spot the common logical errors that TMUA examiners love to test
5. Apply these skills to actual TMUA-style questions

Let's begin.

2 The Master Key: Converting to "If. . . Then"

Here's the single most important idea in this guide:

Key Insight

Almost every unfamiliar-sounding statement in Paper 2—"necessary", "sufficient", "only if", "converse", "contrapositive"—can be converted into a simple "if. . . then" statement. Once you make this conversion, the question becomes straightforward.

Think of "if. . . then" as a *universal translator*. No matter how the examiner phrases a question, you can decode it into this standard form.

2.1 The Arrow Notation

Throughout this guide, we'll use a simple arrow to represent "if. . . then" statements:

$$A \to B \quad \text{means} \quad \text{“If } A\text{, then } B\text{”}$$

That's it. This is the only symbol you need to know. It just saves us writing "if. . . then" repeatedly.

We'll always assume A and B are simple statements like "n is even" rather than whole sentences with quantifiers—we'll add the "for all n" or "there exists n" separately when needed.

Using the Arrow

The statement "If it is raining, then the ground is wet" becomes:

$$\text{Raining} \to \text{Ground wet}$$

2.2 Why This Works

The TMUA specification explicitly states that Paper 2 tests your understanding of statements like:

- "If A then B"
- "A if B"
- "A only if B"
- "A if and only if B"

And concepts like "necessary", "sufficient", "converse", and "contrapositive".

Every single one of these can be written as an "if. . . then" statement (or a pair of them). That's why mastering this translation is the key to Paper 2.

3 The Four Conditional Statements

The TMUA tests four different ways of connecting two statements A and B. Each one has a precise meaning, and confusing them is a common source of errors.

3.1 "If A then B"

This is the most direct form. It says: *whenever A is true, B must also be true.*

Translation Rule

"If A then B" $\iff$ $A \to B$

If. . . Then

Statement: "If n is divisible by 6, then n is divisible by 3."
Translation: (n divisible by 6) $\to$ (n divisible by 3)
Meaning: Whenever you have a number divisible by 6, it must be divisible by 3. (True!)

3.2 "A if B"

This one trips people up. The word "if" introduces the *condition*, so "A if B" means B is the condition for A.

Translation Rule

"A if B" $\iff$ $B \to A$

Watch out: This is the **reverse** of what you might expect!

A if B

Statement: "n is a multiple of 3 if n is a multiple of 6."
Translation: (n is a multiple of 6) $\to$ (n is a multiple of 3)
Why: The word "if" tells you what the condition is. Being a multiple of 6 is the condition; being a multiple of 3 is the consequence.

3.3 "A only if B"

This is perhaps the trickiest. "Only if" is restrictive—it says A can *only* happen when B happens.

Translation Rule

"A only if B" $\iff$ $A \to B$

Think: "The **only** way A can be true is **if** B is true."

Only If

Statement: "n is divisible by 10 only if n is divisible by 5."
Translation: (n divisible by 10) $\to$ (n divisible by 5)
Meaning: If n is divisible by 10, it must be divisible by 5. (Being divisible by 5 is necessary for divisibility by 10.)

3.4 "A if and only if B"

This combines both directions. It means A and B always have the same truth value. We write $A \leftrightarrow B$ as shorthand.

Translation Rule

"A if and only if B" $\iff$ $(A \to B)$ **and** $(B \to A)$

We write this as $A \leftrightarrow B$, meaning both directions hold.

We'll see later that this means A is both necessary and sufficient for B.

If and Only If

Statement: "n^2 is even if and only if n is even."
Translation: Two statements:

- If n^2 is even, then n is even
- If n is even, then n^2 is even

Both are true, so the "if and only if" statement is true.

3.5 Quick Reference Table

English Statement	Arrow Form
If A then B	$A \to B$
A if B	$B \to A$
B if A	$A \to B$
A only if B	$A \to B$
A if and only if B	$A \leftrightarrow B$ (both directions)

The "If" vs "Only If" Trap

Students often confuse these:

- "A **if** B" means $B \to A$ (B is the condition)
- "A **only if** B" means $A \to B$ (A implies B)

They point in **opposite directions**! Always identify which is the condition and which is the consequence.

4 Necessary and Sufficient Conditions

These terms cause more confusion than any other in Paper 2. But once you convert them to "if...then" form, they become completely manageable.

4.1 Sufficient Conditions

A **sufficient condition** is one that *guarantees* a result. If you have a sufficient condition, that's *enough*—you don't need anything else.

Sufficient Condition

"A is **sufficient** for B" $\iff$ $A \to B$

Memory aid: *Sufficient* means "*enough*". If A is enough for B, then having A guarantees B.

Sufficient Condition

Claim: "Being divisible by 6 is sufficient for being divisible by 3."
Translation: (Divisible by 6) $\to$ (Divisible by 3)
Is it true? Yes. Every multiple of 6 is automatically a multiple of 3.
Note: Being divisible by 6 isn't the *only* way to be divisible by 3 (e.g., 9 is divisible by 3 but not 6). But it *guarantees* divisibility by 3.

4.2 Necessary Conditions

A **necessary condition** is one that *must* be true for a result to hold. Without a necessary condition, the result is impossible.

Necessary Condition

"A is **necessary** for B" $\iff$ $B \to A$

Memory aid: *Necessary* means "*needed*". If A is needed for B, then having B means you must have A.

Critical: Note the direction—the arrow goes **from** B **to** A, not from A to B!

Necessary Condition

Claim: "Being divisible by 3 is necessary for being divisible by 6."
Translation: (Divisible by 6) $\to$ (Divisible by 3)
Is it true? Yes. You cannot be divisible by 6 without being divisible by 3.
Note: Being divisible by 3 doesn't *guarantee* divisibility by 6 (e.g., 9). But it's *required* for divisibility by 6.

4.3 The Relationship Between Them

Here's a crucial observation:

Key Insight

"A is sufficient for B" and **"B is necessary for A"** mean **exactly the same thing**. Both translate to: $A \to B$

Concrete example:

- "Being a multiple of 8 is *sufficient* for being a multiple of 4"
- "Being a multiple of 4 is *necessary* for being a multiple of 8"

These say exactly the same thing: (multiple of 8) $\to$ (multiple of 4).

4.4 Worked Example: Identifying Conditions

TMUA-Style Question

Consider the statement: "For a positive integer n, being a perfect square is necessary for n to be a perfect fourth power."

Step 1: Identify the conditions.

- Condition A: n is a perfect square
- Condition B: n is a perfect fourth power

Step 2: Translate using the rule.
"A is necessary for B" translates to $B \to A$.
So: (n is a perfect fourth power) $\to$ (n is a perfect square)
Step 3: Check if it's true.
If $n = k^4$ for some integer k, then $n = (k^2)^2$, so n is a perfect square. ✓
The statement is **true**.

4.5 Necessary and Sufficient Together

When a condition is **both** necessary and sufficient, we have an "if and only if" relationship:

Necessary and Sufficient

"A is **necessary and sufficient** for B" $\iff$ $A \leftrightarrow B$
This means both:

- $A \to B$ (sufficient: A guarantees B)
- $B \to A$ (necessary: B requires A)

4.6 Quick Reference: Necessary vs Sufficient

Statement	**Arrow**	**Meaning**
A is sufficient for B	$A \to B$	A guarantees B
A is necessary for B	$B \to A$	Without A, no B
A is necessary and sufficient for B	$A \leftrightarrow B$	A and B are equivalent

The Direction Trap

The most common error: writing the arrow the wrong way for necessary conditions.
Remember: "A is necessary for B" means $B \to A$, **not** $A \to B$.
If in doubt, think: "Without A, can I have B?" If no, then having B forces you to have A, so $B \to A$.

5 Converse and Contrapositive

Given any "if...then" statement, there are two related statements you need to know: the **converse** and the **contrapositive**.

5.1 The Converse

The **converse** of a statement swaps the "if" and "then" parts.

Converse

Original: $A \to B$ ("If A then B")
Converse: $B \to A$ ("If B then A")

Critical fact: The converse is **NOT** logically equivalent to the original!

Converse Example

Original: "If it is raining, then the ground is wet." (TRUE)
Converse: "If the ground is wet, then it is raining." (FALSE—could be a sprinkler!)
The original can be true while the converse is false.

5.2 The Contrapositive

The **contrapositive** swaps *and negates* both parts.

Contrapositive

Original: $A \to B$ ("If A then B")
Contrapositive: (not B) $\to$ (not A) ("If not B then not A")

Critical fact: The contrapositive **IS** logically equivalent to the original!

Contrapositive Example

Original: "If it is raining, then the ground is wet." (TRUE)
Contrapositive: "If the ground is not wet, then it is not raining." (ALSO TRUE!)
These two statements say exactly the same thing, just from different angles.

5.3 Why the Contrapositive Works

Think about it this way: if rain *always* makes the ground wet, then finding dry ground *proves* it's not raining. This is the logical basis for **proof by contrapositive**.

Key Insight

To prove $A \to B$, you can instead prove its contrapositive: (not B) $\to$ (not A).
This is often easier, especially when B is simpler to negate than to prove directly.

Proof by Contrapositive

To prove: If n^2 is odd, then n is odd.
Contrapositive: If n is even, then n^2 is even.
Proof of contrapositive: If n is even, write $n = 2k$. Then $n^2 = 4k^2 = 2(2k^2)$, which is even. ✓
Since the contrapositive is true, the original statement is also true.
This is much easier than proving the original directly!

5.4 Summary Table

Starting with $A \to B$ ("If A then B"):

Name	Statement	Equivalent to Original?
Original	$A \to B$	—
Converse	$B \to A$	**NO**
Contrapositive	(not B) $\to$ (not A)	**YES**

The Converse Trap

A very common TMUA error: assuming that because $A \to B$ is true, $B \to A$ must also be true.
Example of this error:

- "All squares are rectangles" is true (Square $\to$ Rectangle)
- "All rectangles are squares" is false (the converse)

When spotting errors in proofs, look for this mistake!

6 "For All" and "There Exists"

Paper 2 frequently tests statements involving **"for all"** (meaning *every* case) and **"there exists"** (meaning *at least one* case).

6.1 Understanding "For All"

A "for all" statement claims something is true in *every* case without exception.

For All

"For all integers n, $n^2 \geq 0$."
Meaning: Every single integer, when squared, gives a non-negative result.
To prove it: Show it works for *any* integer.
To disprove it: Find *one* counterexample. A single exception is enough to break a "for all" statement.

6.2 Understanding "There Exists"

A "there exists" statement claims something is true in *at least one* case.

There Exists

"There exists an integer n such that $n^2 = n$."
Meaning: At least one integer equals its own square.
To prove it: Find *one* example (e.g., $n = 0$ or $n = 1$).
To disprove it: Show it's impossible for *any* integer.

6.3 "For Some"

The phrase "for some" means the same as "there exists"—at least one case exists.

Translation Rule

"For some x, $P(x)$" $\iff$ "There exists x such that $P(x)$"

6.4 Important Patterns

Many TMUA questions combine quantifiers with conditional statements:

Statement	**Meaning**
For all x, if $P(x)$ then $Q(x)$	Every x satisfying P also satisfies Q
There exists x such that $P(x)$ and $Q(x)$	At least one x satisfies both P and Q
For all x, $P(x)$ or $Q(x)$	Every x satisfies at least one of P or Q

Quantifiers with Conditionals

Statement: "For all real x, if $x > 2$, then $x^2 > 4$."
Meaning: Every real number greater than 2 has a square greater than 4.
Compare with: "There exists a real x such that $x > 2$ and $x^2 \leq 4$."
This would be the negation—and it's false (there's no such x), confirming the original is true.

7 Negating Statements

Negation is a high-yield skill for Paper 2. You need to know how to negate statements involving "and", "or", conditionals, and quantifiers.

7.1 Important: "Or" Means "And/Or"

Key Insight

In TMUA logic questions, the word **"or"** always means **inclusive or** (and/or).
A statement "A or B" is true if:

- A is true (and B may or may not be true), or
- B is true (and A may or may not be true), or
- Both A and B are true

The only way "A or B" is *false* is if *both* A and B are false.

7.2 Negating "And" and "Or"

These rules might seem counterintuitive at first, but they make perfect sense once you think about them:

Negating And/Or

Not (A **and** B) $\iff$ (**not** A) **or** (**not** B)
Not (A **or** B) $\iff$ (**not** A) **and** (**not** B)

Memory aid: When you negate, "and" becomes "or" and vice versa.

Negating And

Original: "$x > 0$ and $x < 10$" (i.e., x is between 0 and 10)
Negation: "$x \leq 0$ or $x \geq 10$" (i.e., x is outside the interval)
Why: To *fail* to be in the interval, x must fail at least one condition.

Negating Or

Original: "n is even or n is a multiple of 3"
Negation: "n is odd and n is not a multiple of 3"
Why: To *fail* "A or B", you must fail *both*.
Examples: $n = 5, 7, 11, 13, \ldots$ satisfy the negation.

7.3 Negating Conditionals

This one surprises many students:

Negating If. . . Then

Not $(A \to B) \iff A$ **and not** B

Why: The *only* way "if A then B" can fail is when A is true but B is false.
If A is false, the whole "if A then B" statement counts as true—this is called **vacuous truth**.

Negating a Conditional

Original: "If n is divisible by 4, then n is even."
Negation: "n is divisible by 4 and n is not even."
Note: This negation is actually false (no such n exists), confirming the original is true.

7.4 Negating Quantifiers

This is crucial for Paper 2:

Negating Quantifiers

Not (for all x, $P(x)$) $\iff$ There exists x such that **not** $P(x)$
Not (there exists x such that $P(x)$) $\iff$ For all x, **not** $P(x)$

Memory aid: "For all" becomes "there exists" and vice versa, then negate the property.

Negating For All

Original: "For all real numbers x, $x^2 > 0$."
Negation: "There exists a real number x such that $x^2 \leq 0$."
Is the negation true? Yes! Take $x = 0$. So the original statement is **false**.

Negating There Exists

Original: "There exists an integer n such that $n^2 = 2$."
Negation: "For all integers n, $n^2 \neq 2$."
Is the negation true? Yes (since $\sqrt{2}$ is irrational). So the original is **false**.

7.5 Negation Summary Table

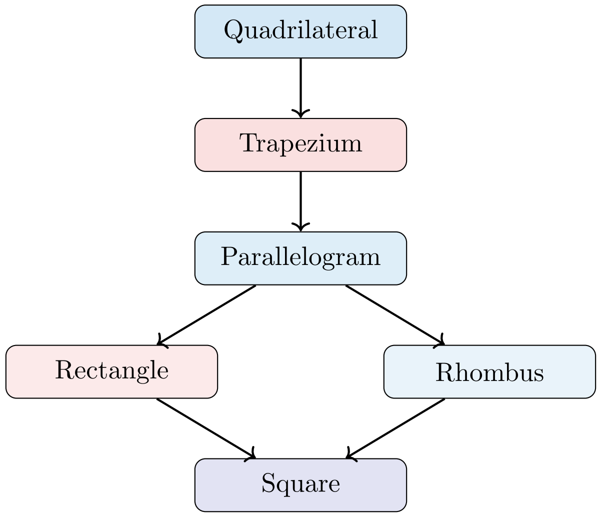
Original Statement	Negation
A and B	(not A) or (not B)
A or B	(not A) and (not B)
If A then B	A and (not B)
For all x, $P(x)$	There exists x such that not $P(x)$
There exists x with $P(x)$	For all x, not $P(x)$

8 Geometry: The Art of Counterexamples

Geometry questions in Paper 2 often ask whether certain properties are necessary or sufficient for a shape to have a particular classification. The key strategy is to use **counterexamples**—specific shapes that break the claimed relationship.

8.1 The Quadrilateral Family

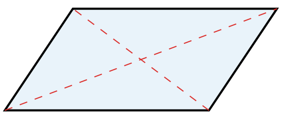
Understanding the relationships between quadrilaterals is essential. Here's the hierarchy:



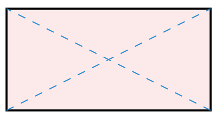
Read upward: A square is a special case of both a rectangle and a rhombus, which are both special cases of a parallelogram, etc.

8.2 Key Counterexample Shapes

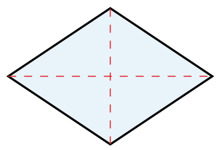
When disproving claims about quadrilaterals, these shapes are your best friends:



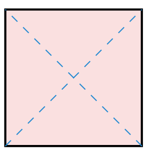
Parallelogram
Unequal diagonals



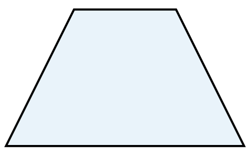
Rectangle
Equal diagonals, 90° angles



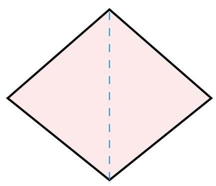
Rhombus
All sides equal, not 90°



Square
All properties combined



Trapezium
Only one pair parallel



Kite
Two pairs adjacent equal

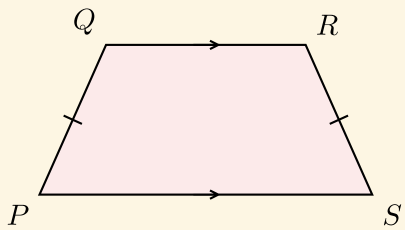
8.3 Using Counterexamples

Necessary Condition Counterexample

Claim: "Having equal diagonals is necessary for being a parallelogram."
Translation: Parallelogram → Equal diagonals
To disprove: Find a parallelogram *without* equal diagonals.
Counterexample: A skewed parallelogram (not a rectangle) has unequal diagonals.
Conclusion: The claim is **FALSE**. Equal diagonals are *not* necessary.

Sufficient Condition Counterexample

Claim: "Having one pair of equal sides and one pair of parallel sides is sufficient for being a parallelogram."
Translation: (Equal pair + Parallel pair) → Parallelogram
To disprove: Find a shape with these properties that is *not* a parallelogram.
Counterexample: A trapezium can have $PQ = SR$ (equal) and $QR \parallel PS$ (parallel), but still not be a parallelogram.



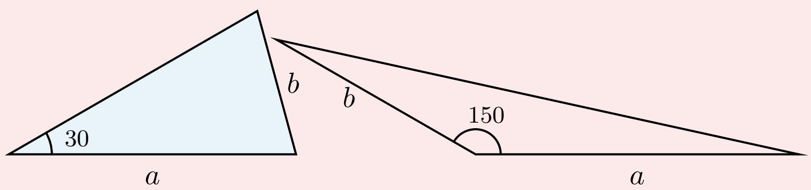
Conclusion: The claim is **FALSE**. These conditions are *not* sufficient.

8.4 The Triangle Congruency Trap (SSA)

TMUA loves testing whether certain conditions guarantee triangle congruency. The infamous **SSA (Side-Side-Angle)** case is *not* generally sufficient.

The Ambiguous Case

SSA does NOT prove congruency in general, because two different triangles can share two sides and a non-included angle.
Why? The formula Area $= \frac{1}{2}ab\sin(C)$ and the fact that $\sin(\theta) = \sin(180 - \theta)$ means two different angles can give the same area.



Same a, b, and area!

Exception: SSA *does* work when the angle is 90 (RHS: Right-angle, Hypotenuse, Side).

8.5 Counterexample Strategy: Try Negatives!

Key Insight

When testing inequalities involving variables, **always try negative numbers** as counterexamples.
Many "obvious" statements fail when negatives are involved:

- $a^2 > a$ is FALSE when $a = 0.5$ or $a = -1$
- $a < b$ implies $a^2 < b^2$ is FALSE when $a = -3, b = 1$
- $|a + b| = |a| + |b|$ is FALSE when $a = 1, b = -1$

9 Spotting Errors in Proofs

Paper 2 frequently presents "proofs" containing deliberate errors and asks you to identify the flaw. Here are the most common invalid deductions to watch for.

9.1 Common Error 1: Assuming the Converse

Invalid: Assuming the Converse

The error: Assuming that because $A \to B$ is true, $B \to A$ is also true.
Example:

> "n^2 is divisible by 4, so n is divisible by 4."

Why it's wrong: $n = 2$ gives $n^2 = 4$, which is divisible by 4, but $n = 2$ is not divisible by 4.
The statement "n divisible by $4 \Rightarrow n^2$ divisible by 4" is true, but its converse is false.

9.2 Common Error 2: Dividing Without Checking for Zero

Invalid: Division by Zero

The error: Claiming that "if $ab = ac$, then $b = c$".
The rule: From $ab = ac$, you can only cancel a when you already know $a \neq 0$.
Example:

> "We have $x(x-1) = x \cdot 2$, so $x - 1 = 2$, giving $x = 3$."

Why it's wrong: This ignores the case $x = 0$, where both sides equal 0.

9.3 Common Error 3: Assuming Functions are Injective

Invalid: Non-Injective Functions

The error: Claiming that "if $f(A) = f(B)$, then $A = B$".
This is only valid if f is *injective* (one-to-one)—meaning it never hits the same output twice.
Graphical test: If the graph of $y = f(x)$ ever hits the same y-value twice (fails the horizontal line test), then f is not injective.
Example:

> "$\sin A = \sin B$, therefore $A = B$."

Why it's wrong: $\sin(30) = \sin(150) = 0.5$, but $30 \neq 150$.
Similar traps: $x^2 = y^2$ does not imply $x = y$ (could have $x = -y$).

9.4 Common Error 4: Wrong Direction in Proof

Invalid: Proving the Wrong Direction

The error: When asked to prove $A \to B$, instead proving $B \to A$.
Example:

To prove: "If n^2 is even, then n is even."

Flawed proof: "If n is even, say $n = 2k$, then $n^2 = 4k^2$ which is even."

Why it's wrong: This proves the converse, not the original statement.

9.5 Common Error 5: Invalid Algebraic Manipulation

Invalid: Square Root Errors

The error: Forgetting that $\sqrt{x^2} = |x|$, not x.
Example:

"$x^2 = 9$, so $x = \sqrt{9} = 3$."

Why it's wrong: $x = -3$ is also a solution.

9.6 Checklist for Error-Spotting Questions

When presented with a "proof" to check, ask yourself:

1. Has the argument assumed the converse of something?
2. Has any division occurred? Was zero excluded?
3. Has any function been treated as injective (one-to-one) when it isn't?
4. Is the proof actually proving what was asked, or the converse?
5. Have any cases been forgotten (e.g., negative numbers, zero)?
6. Have square roots been handled correctly?

10 Practice Questions

Test yourself with these five TMUA-style questions. Try them all before checking the solutions on the following pages.

Question 1: Necessary & Sufficient

Which of the following is equivalent to the statement "Being a multiple of 4 is necessary for being a multiple of 8"?

A. If n is a multiple of 4, then n is a multiple of 8

B. If n is a multiple of 8, then n is a multiple of 4

C. n is a multiple of 4 if and only if n is a multiple of 8

D. If n is not a multiple of 4, then n is a multiple of 8

Question 2: Negation

Consider the statement P: "For all real numbers x, if $x^2 < 4$ then $x < 2$."
Which of the following is the negation of P?

A. For all real numbers x, if $x^2 \geq 4$ then $x \geq 2$

B. There exists a real number x such that $x^2 < 4$ and $x \geq 2$

C. There exists a real number x such that $x^2 \geq 4$ and $x < 2$

D. For all real numbers x, $x^2 \geq 4$ or $x < 2$

Question 3: Proof Analysis

The following is a proposed proof that all positive integers are even:

> *Let $P(n)$ be the statement "n is even".*
>
> *Base case: $P(2)$ is true since $2 = 2 \times 1$.*
>
> *Inductive step: Assume $P(k)$ is true for some $k \geq 2$. Then $k = 2m$ for some integer m. Now $k + 2 = 2m + 2 = 2(m + 1)$, so $P(k + 2)$ is true.*
>
> *By induction, $P(n)$ is true for all positive integers n.*

What is the error in this proof?

A. The base case is wrong

B. The inductive step is wrong

C. The base case doesn't cover all required starting points

D. Induction cannot be used for this type of statement

Question 4: Converse

Let n be a positive integer. Consider the statement:
"n being odd is sufficient for $n^2 + n$ being even."
Which of the following is true?

A. The statement is true, and its converse is also true

B. The statement is true, but its converse is false

C. The statement is false, but its converse is true

D. The statement is false, and its converse is also false

Question 5: Geometry

Consider the following statements about a quadrilateral $ABCD$:
Statement I: Having two pairs of equal opposite sides is sufficient for $ABCD$ to be a parallelogram.
Statement II: Having diagonals that bisect each other is necessary for $ABCD$ to be a parallelogram.
Which of the following is correct?

A. Both statements are true

B. Statement I is true, Statement II is false

C. Statement I is false, Statement II is true

D. Both statements are false

Solutions

Solution to Question 1

Step 1: Identify the structure.

- A = "n is a multiple of 4"
- B = "n is a multiple of 8"

Step 2: Apply the translation rule.
"A is necessary for B" translates to $B \to A$.
This is: "If n is a multiple of 8, then n is a multiple of 4."
Answer: B
Verification: Every multiple of 8 (8, 16, 24, 32, ...) is indeed a multiple of 4. ✓

Solution to Question 2

Step 1: Write P in structured form.
P: For all x, $(x^2 < 4 \to x < 2)$
Step 2: Negate step by step.
Negating "for all" gives "there exists", and we negate the inner statement:
Not P: There exists x such that not$(x^2 < 4 \to x < 2)$
Step 3: Negate the conditional.
Not $(A \to B) = A$ and not B
So: not$(x^2 < 4 \to x < 2) = (x^2 < 4)$ and $(x \geq 2)$
Step 4: Combine.
Not P: There exists x such that $x^2 < 4$ and $x \geq 2$.
Answer: B
Note: There is no real x with $x^2 < 4$ and $x \geq 2$, so the negation is false. Therefore the original statement P is true.

Solution to Question 3

Step 1: Check the base case.
$P(2)$ claims 2 is even. This is true. The base case itself is correct.
Step 2: Check the inductive step.
The step shows $P(k) \to P(k+2)$. This is valid: if k is even, so is $k+2$.
Step 3: Identify the flaw.
The induction only proves $P(n)$ for $n = 2, 4, 6, 8, \ldots$ (even numbers).
It never proves $P(1)$, $P(3)$, $P(5)$, etc.
To prove the statement for *all* positive integers, we would need:

- Either a base case at $P(1)$ with step $P(k) \to P(k+1)$, or
- Base cases at both $P(1)$ and $P(2)$ if using step $P(k) \to P(k+2)$

The proof only has base case $P(2)$, so it misses all odd integers.
Answer: C

Solution to Question 4

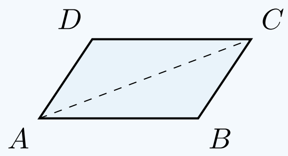
Step 1: Translate the statement.
"n odd is sufficient for $n^2 + n$ even" means:
(n is odd) $\to$ ($n^2 + n$ is even)
Step 2: Check if the statement is true.
If n is odd, write $n = 2k + 1$. Then:

$$\begin{aligned} n^2 + n &= n(n+1) \\ &= (2k+1)(2k+2) \\ &= 2(2k+1)(k+1) \end{aligned}$$

This is even. So the statement is **true**.
Step 3: State and check the converse.
Converse: ($n^2 + n$ is even) $\to$ (n is odd)
Note that $n^2 + n = n(n + 1)$, which is the product of two consecutive integers.
One of n and $n + 1$ is always even, so $n(n + 1)$ is *always* even.
Counterexample: $n = 2$ gives $n^2 + n = 6$, which is even, but $n = 2$ is not odd.
So the converse is **false**.
Answer: B

Solution to Question 5

Analyse Statement I:
"Two pairs of equal opposite sides is sufficient for parallelogram" means:
(Two pairs equal opposite sides) $\to$ Parallelogram
Is this true? Yes! If $AB = CD$ and $BC = DA$, the quadrilateral must be a parallelogram. This can be proved by drawing a diagonal and using SSS congruence.



Statement I is **TRUE**.
Analyse Statement II:
"Diagonals bisecting each other is necessary for parallelogram" means:
Parallelogram $\to$ Diagonals bisect each other
Is this true? Yes! In any parallelogram, the diagonals always bisect each other. This is a standard property.
Statement II is **TRUE**.
Answer: A (Both statements are true)

11 Quick Reference Sheet

Tear this out (or save it) for exam day.

Conditional Statement Translations

English	**Arrow Form**
If A then B	$A \to B$
A if B	$B \to A$
B if A	$A \to B$
A only if B	$A \to B$
A if and only if B	$A \leftrightarrow B$

Note: $A \leftrightarrow B$ means "A if and only if B" (both $A \to B$ and $B \to A$).

Necessary and Sufficient

Statement	**Translation**
A is sufficient for B	$A \to B$
A is necessary for B	$B \to A$
A is necessary & sufficient for B	$A \leftrightarrow B$

Converse and Contrapositive

Starting with: $A \to B$

Name	**Form**	**Equivalent?**
Converse	$B \to A$	**NO**
Contrapositive	(not B) $\to$ (not A)	**YES**

Negation Rules

To negate...	**Becomes...**
A and B	(not A) or (not B)
A or B	(not A) and (not B)
$A \to B$	A and (not B)
For all x, $P(x)$	There exists x with not $P(x)$
There exists x with $P(x)$	For all x, not $P(x)$

Common Errors to Spot

1. **Assuming the converse:** $A \to B$ does NOT mean $B \to A$
2. **Dividing by zero:** $ab = ac$ only gives $b = c$ when $a \neq 0$
3. **Non-injective functions:** $\sin A = \sin B$ does NOT mean $A = B$
4. **Square root errors:** $\sqrt{x^2} = |x|$, not x
5. **Wrong direction:** Check you're proving $A \to B$, not $B \to A$

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